Appearance Models

Stephen Pizer

Appearance Models

- Data match term, w/ images as data
  - For segmentation
  - For registration
- Objectives
  - Represent actual differences in the population
  - Yield geometry-to-image match that
    - has few local minima
    - has long capture distance

Appearance Models

- Intensities in voxels in region
- Derivatives in voxels in region
- Histograms in regions
- Quantile functions
    - Of intensities
    - Of derived features
    - Of tuples of features

Geometry-to-Image Match Measures for relevant region

- Sum of squared differences over voxels
  - Only when same image type and calibrated
- Normalized correlation
- Mutual information: Info by one intensity on the other
  - Tightness of scattergram (over pixels) of I_1 vs I_2
  - Especially when different image type
  - Variant: normalized mutual information
  - Harder to compute than sum of squared differences
- Histograms
  - Sum of squared distances
  - Earthmover's distance: see quantile functions
- Quantile functions
    - Regional features
- Mahalanobis distances after PCA training

Geometry-to-Image Match Measures by sum of squared intensity differences

- Common, but only good trait is mathematical ease
  - Justified only if differences between image are just imaging noise
    - So only for within-patient variation
    - Requires careful intensity normalization or calibration
      - E.g., not good for fan-beam CT vs. cone-beam CT
  - Focus is equal over voxels but needs to focus on voxels especially important to the object shape
- Normalized correlation is similarly flawed

Mutual Information: when image intensities are incomparable

Mutual information is the amount of uncertainty about one variable (here the target image intensities) cleared up by knowing the other (here the ref image intensities). Alternatively,

Mutual information is the entropy difference of observing two random variables jointly or separately, i.e.,

$$
\begin{aligned}
I[\mathbf{x};\mathbf{y}] &:= H[\mathbf{x}] - H[\mathbf{x}|\mathbf{y}] \\
&= H[\mathbf{y}] - H[\mathbf{y}|\mathbf{x}] \\
&= H[\mathbf{x}] + H[\mathbf{y}] - H[\mathbf{x},\mathbf{y}].
\end{aligned}
$$

The mutual information is zero iff **x** and **y** are independent.

Then

$$p(\mathbf{x},\mathbf{y}) = p(\mathbf{x})p(\mathbf{y}).$$

Entropy

Given a discrete random variable X with outcomes x_i, $i = 1, 2, ..., n$ the Shannon information entropy is defined as

$$H[X] := -\sum_{i=1}^{n} p(x_i) \log p(x_i).$$

What's the intuition behind this?

Define the level of surprise of observing x_i as $s_i = -\log p(x_i)$.
An observation of x_i which has a small probability is very surprising!

Then, $H[x]$ is simply the expected value of S, i.e.

$$E[S] = \sum_{i=1}^{n} s_i p(x_i) = H[X].$$

Differential Entropy

The differential entropy for a random variable $\mathbf{x}$ with pdf $p(\mathbf{x})$ is

$$H[\mathbf{x}] := -\int p(\mathbf{x}) \log p(\mathbf{x}) \; d\mathbf{x}.$$

The joint entropy of two random variables $\mathbf{x}$ and $\mathbf{y}$ is

$$H[\mathbf{x}, \mathbf{y}] = -\int p(\mathbf{x}, \mathbf{y}) \log p(\mathbf{x}, \mathbf{y}) \; d\mathbf{x} \; d\mathbf{y}.$$

Their conditional entropy is

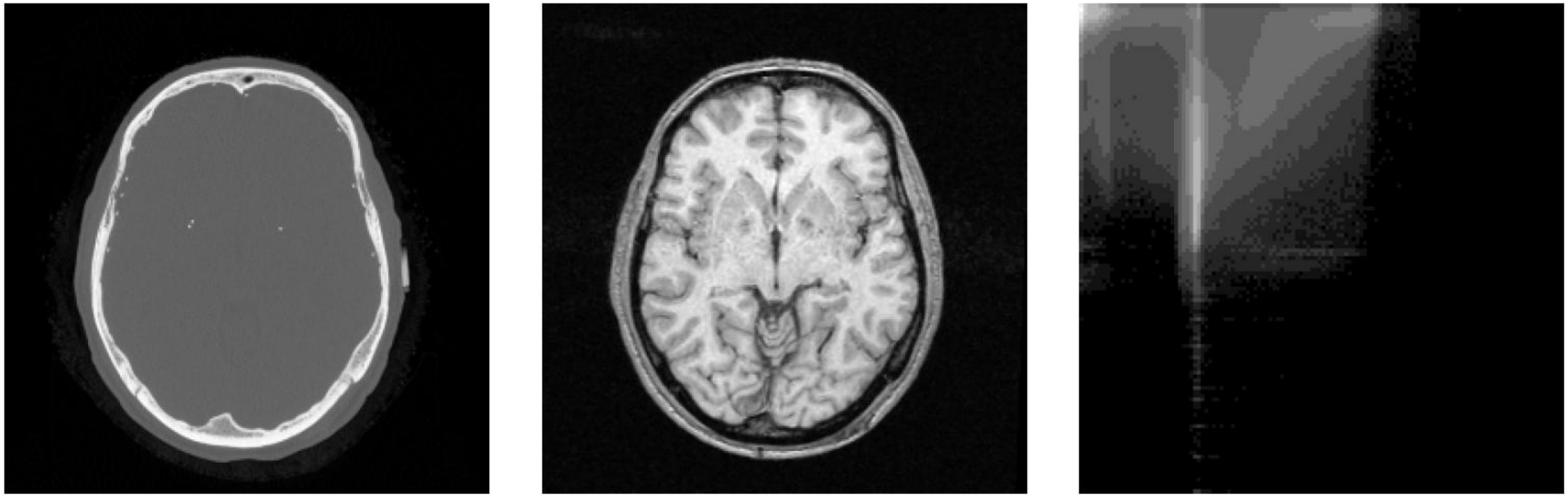
$$H[\mathbf{x}|\mathbf{y}] = -\int p(\mathbf{x}, \mathbf{y}) \log p(\mathbf{x}|\mathbf{y}) \; d\mathbf{x} \; d\mathbf{y}.$$

It follows

$$H[x, y] = H[y|x] + H[x] = H[x|y] + H[y].$$

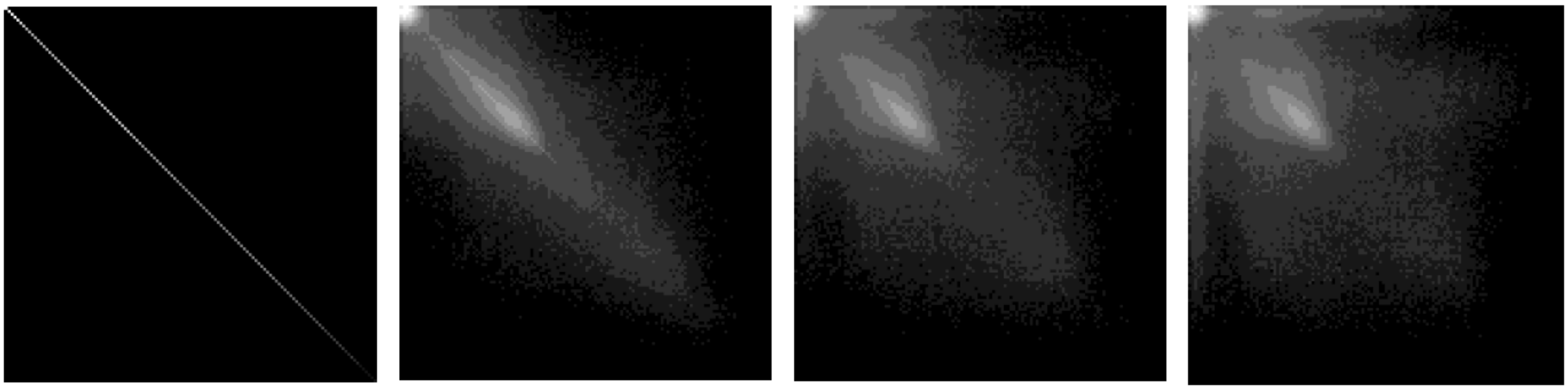
Mutual Information

Here, outcomes are I_1, I_2 in relevant pixels



Images: Pluim et al.

Mutual Information



3.82 Image with itself

6.79 Rotated, 2 degrees

6.98 Rotated, 5 degrees

7.15 Rotated, 10 degrees

Numbers shown are joint entropy:
Mutual information here is const. $[H(I_1) + H(I_2)]$ – number shown

Alignment causes clustered histogram and low entropy, so high mutual information.

Images: Pluim et al.

Quantile Functions

- Quantile functions: a representation of histograms that allows efficient computation of differences [RE Broadhurst dissertation; on midag.cs.unc.edu]
  - Inverse of cumulative histogram: feature value vs. percentile

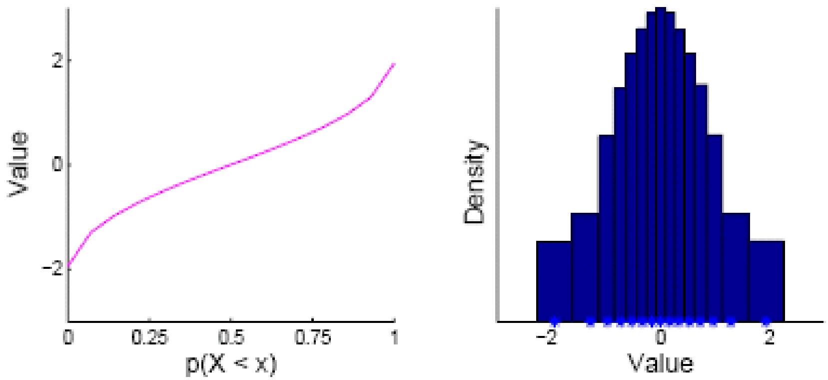


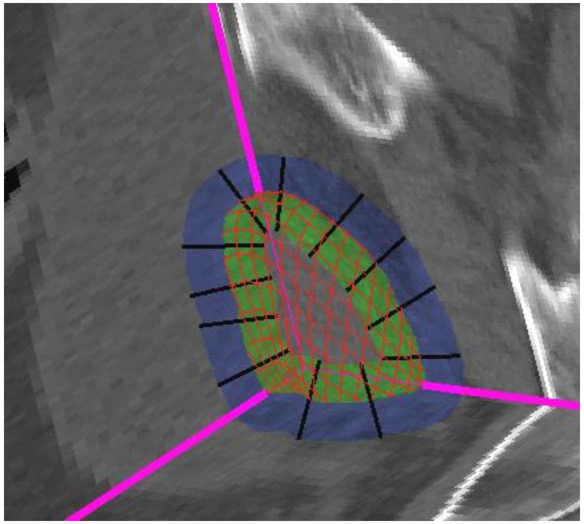
Figure 2.2: The Gaussian distribution represented as (left) a discrete quantile function with 25 values and (right) the QF's corresponding adaptive bin histogram.

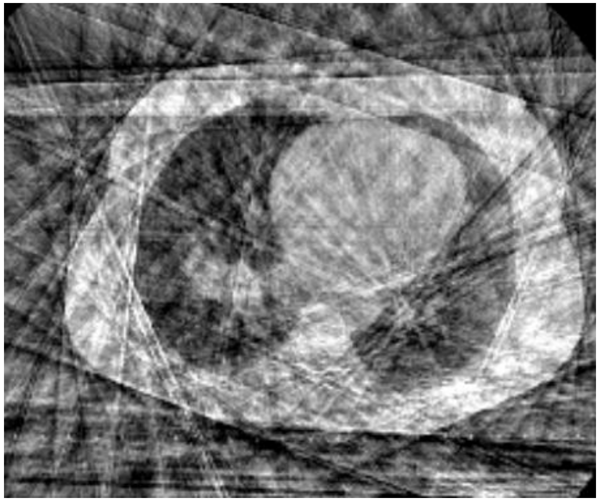
Quantile Functions for Appearance Models Relative to Object Models

- A natural distance between histograms: the earth-mover's distance
  - Euclidean difference (QF_1, QF_2) = Earth-mover's distance ($Histogram_1$, $Histogram_2$)
- PCA on QFs supported by this Euclidean difference
  - Yields $-\log p(I \mid z)$ as Mahalanobis distance
    - Probability distributions trainable with relatively few training cases
  - Or use ordinary Euclidean difference

Features for Quantile Functions for Appearance Models

- Intensity in a region
- Texture features in a region
- Distance to tissue type [confidential within UNC]
  - Allows insensitivity to artifacts
  - Provides long capture distances and smooth objective functions





Appearance Models
Relative to M-reps

- M-rep-relative interrogation of image
  - u = (hub index, side, fraction of spoke)
  - u to x calculation, then interpolate I(x) from nearby voxels
- Quantile functions
  - Euclidean differences
  - Mahalanobis distances